

6th USA/Europe ATM 2005 R&D Seminar

Statistical performance evaluation between linear and nonlinear designs for aircraft relative guidance

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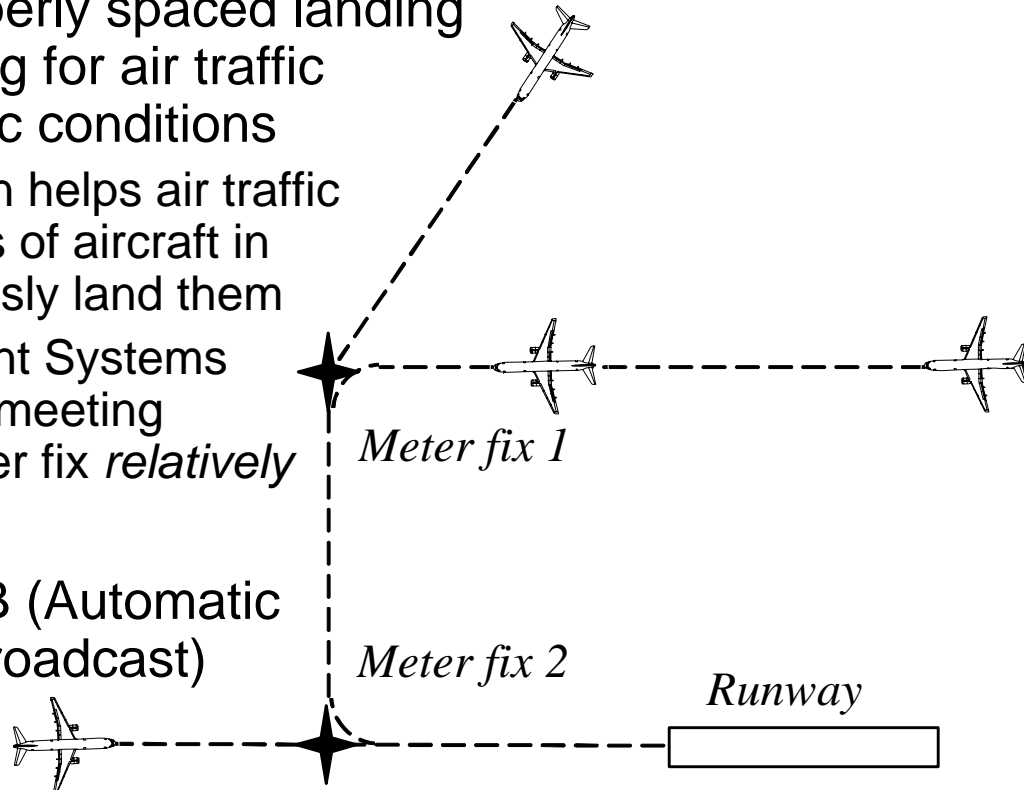
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Overview

- **Introduction**
- Feedback loop design
- Statistical performance evaluation
- Conclusion

Introduction

- Main tasks of air traffic controllers managing arrival traffic
 - to sequence, merge and space aircraft for landing
- The task of establishing properly spaced landing sequences is very demanding for air traffic controllers under heavy traffic conditions
 - Arrival Manager (AMAN) often helps air traffic controllers to build sequences of aircraft in order to safely and expeditiously land them
 - But current Flight Management Systems (FMS) are *not* yet capable of meeting a specified time-lag over meter fix *relatively* to another aircraft
- Enabling technology : ADS-B (Automatic Dependent Surveillance – Broadcast)



Introduction

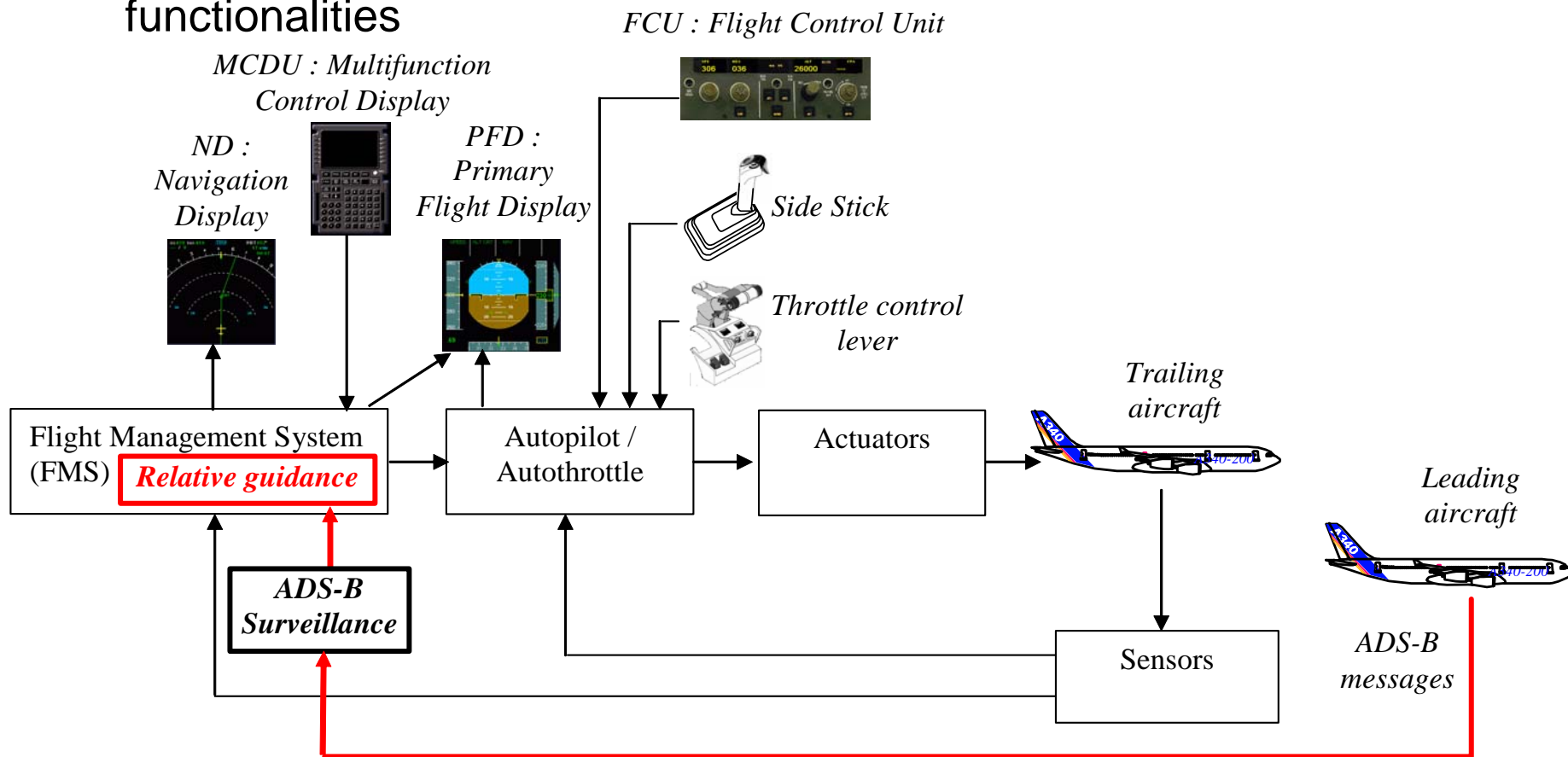
- Scope of the paper
 - Design of a linear and a nonlinear feedback loop for a new functionality onboard aircraft : *relatively* to another aircraft, merging at a specified meter fix and then station keeping
 - Compare performances of both designs through simulations

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Feedback loop design

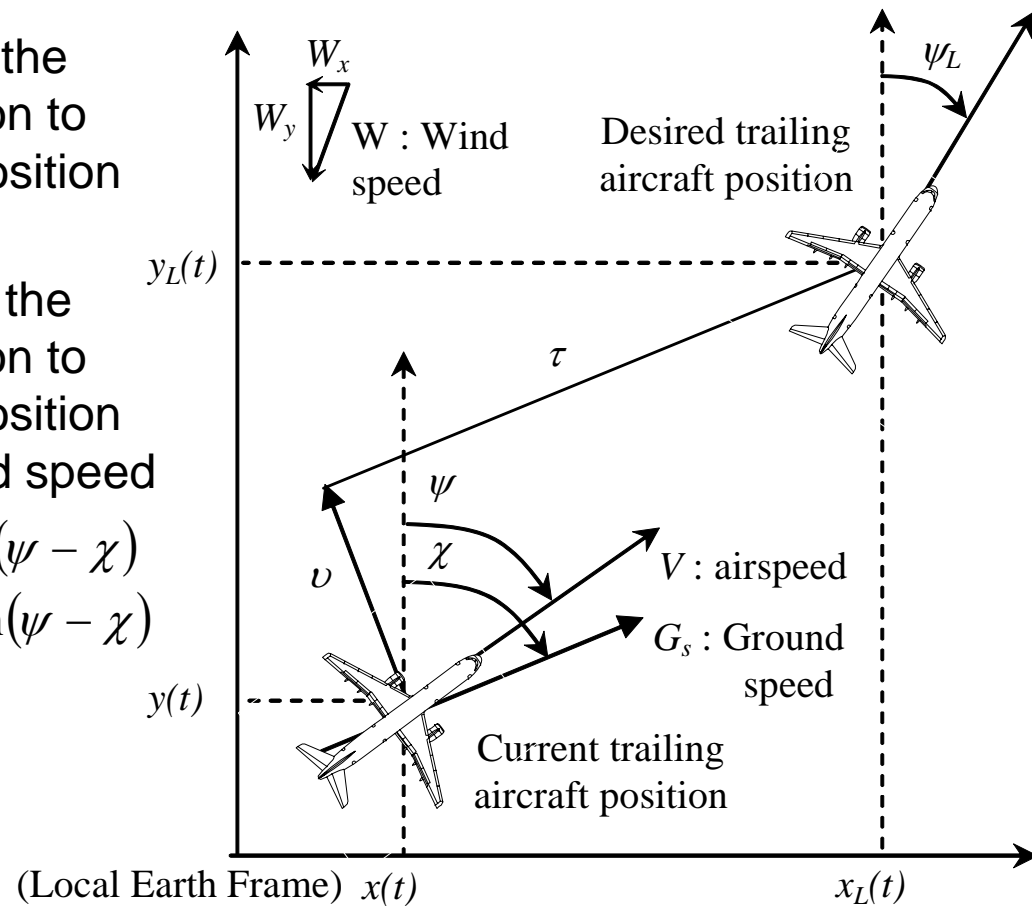
- The relative guidance feedback loop is envisioned as part of FMS functionalities



Feedback loop design

- Relative motion dynamics
 - τ : along track distance from the current trailing aircraft position to the desired trailing aircraft position along the ground speed
 - ν : cross track distance from the current trailing aircraft position to the desired trailing aircraft position perpendicularly to the ground speed

$$\begin{cases} \dot{\tau} = \dot{\chi} \cdot \nu + V_L \cdot \cos(\psi_L - \chi) - V \cdot \cos(\psi - \chi) \\ \dot{\nu} = -\dot{\chi} \cdot \tau + V_L \cdot \sin(\psi_L - \chi) - V \cdot \sin(\psi - \chi) \end{cases}$$



Feedback loop design

- State vector

- Along track τ and cross track distance ν : $\mathbf{x}_1 = [\tau \quad \nu]^T$
- Heading ψ and True Air Speed V : $\mathbf{x}_2 = [\psi \quad V]^T$

- Control vector

- heading rate $d\psi/dt$ and longitudinal acceleration dV/dt : $\mathbf{u} = [\dot{\psi} \quad \dot{V}]^T$

- State space representation

$$\begin{cases} \dot{\mathbf{x}}_1 = A(\mathbf{x}_2, \mathbf{u}) \cdot \mathbf{x}_1 + B(\mathbf{x}_2) \\ \dot{\mathbf{x}}_2 = \mathbf{u} \end{cases} \quad \text{Where} \quad \begin{cases} A(\mathbf{x}_2, \mathbf{u}) = \begin{bmatrix} 0 & \dot{\chi}(\mathbf{x}_2, \mathbf{u}) \\ -\dot{\chi}(\mathbf{x}_2, \mathbf{u}) & 0 \end{bmatrix} \\ B(\mathbf{x}_2) = \begin{bmatrix} G_{sL} \cdot \cos(\chi_L - \chi) - G_s \\ G_{sL} \cdot \sin(\chi_L - \chi) \end{bmatrix} \end{cases}$$

- The relative motion dynamics is basically nonlinear

- The design objective is to render the equilibrium points $\mathbf{x}_{1e} = [0 \quad 0]^T$ and $\mathbf{x}_{2e} = [\psi_L \quad V_L]^T$ globally asymptotically stable

Feedback loop design

- A first alternative to design a feedback loop which stabilizes the relative motion dynamics around equilibrium points \mathbf{x}_{1e} and \mathbf{x}_{2e} consists in linearizing the nonlinear system
- This is clearly a local approach, but it is the standard way to design flight guidance feedback loop
- Using linearized relative motion dynamics, the feedback loop is expressed as a function of the available state vector \mathbf{x}_1 and \mathbf{x}_2

$$\mathbf{u} = -\hat{B}^{-1}(\Lambda_d \cdot \hat{B} \cdot (\mathbf{x}_2 - \mathbf{x}_{2e}) + \Lambda_p \mathbf{x}_1)$$
- The resulting proportional and derivative (*PD*) feedback loop is time varying since matrix \hat{B}^{-1} is not constant and evolves with heading and airspeed of the leading aircraft

$$\hat{B}^{-1} = \begin{bmatrix} \sin(\psi_L - \chi_L)/V_L & -\cos(\psi_L - \chi_L)/V_L \\ -\cos(\psi_L - \chi_L) & -\sin(\psi_L - \chi_L) \end{bmatrix}$$

Feedback loop design

- Nonlinear feedback loop design
 - The system which represents the relative motion dynamics consists of two cascaded systems with state vectors \mathbf{x}_1 and \mathbf{x}_2
 - The matrix $A(\mathbf{x}_2, \mathbf{u})$ is skew-symmetric
 - Vectorial backstepping : for construction of both feedback control law and associated Lyapunov function

$$\mathbf{u} = - \left(\frac{\partial B(\mathbf{x}_2)}{\partial \mathbf{x}_2} \right)^{-1} \left(\left(\frac{K_2}{2} + \Lambda_2 + \Lambda_1 \right) B(\mathbf{x}_2) + \left(\frac{K_2}{2} + \Lambda_2 \right) \Lambda_1 \mathbf{x}_1 \right)$$

- Nonlinear design key points
 - Matrix Λ_1 is chosen as a diagonal matrix
 - Use Young's inequality : $xy \leq (x^2 + y^2)/2$
 - The key point of the nonlinear feedback loop design is that matrix $\frac{\partial B(\mathbf{x}_2)}{\partial \mathbf{x}_2}$ is invertible

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Statistical performance evaluation

- Fair comparison between the linear and the nonlinear designs
 - Matrices K_2 , Λ_1 , Λ_2 , Λ_p and Λ_d have been chosen so that the backstepping feedback loop and the proportional and derivative feedback loop simplify to the same feedback loop around the equilibrium point \mathbf{x}_{2e}

$$\left(\frac{K_2}{2} + \Lambda_2 \right) \Lambda_1 = \Lambda_p \quad \text{and} \quad \frac{K_2}{2} + \Lambda_2 + \Lambda_1 = \Lambda_d$$

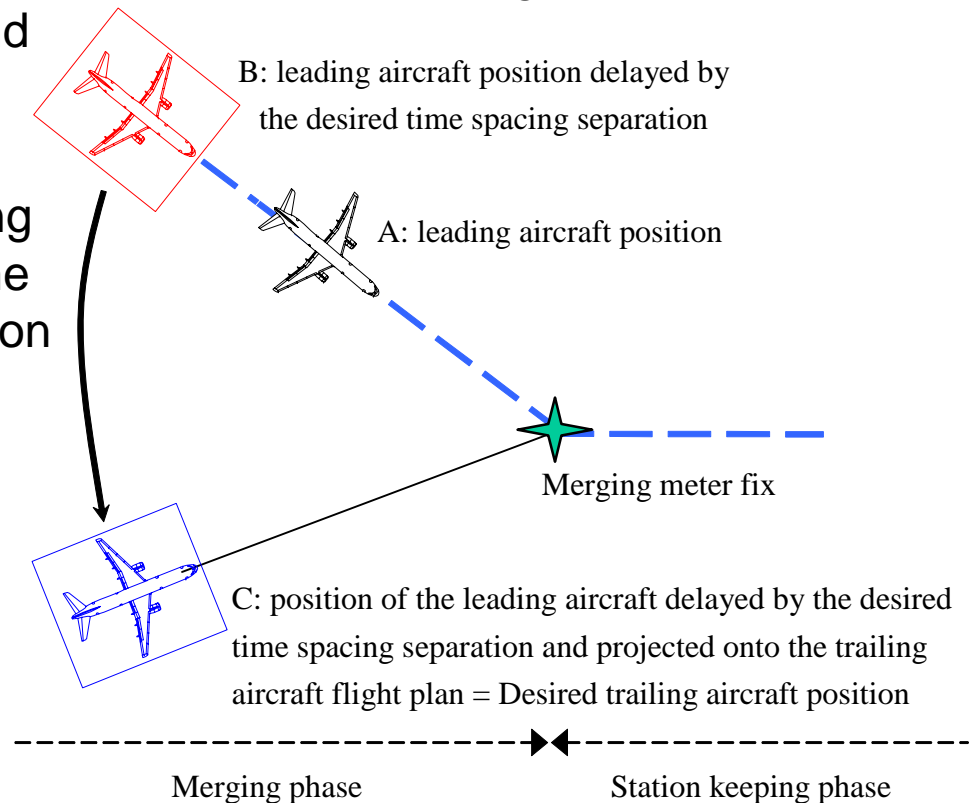
- In order to comply with time responses of the airspeed and bank angle control channels for a wide body aircraft such as an Airbus A320, the matrices Λ_p and Λ_d have been set as follows

$$\Lambda_p = \begin{bmatrix} 4 \cdot 10^{-4} & 0 \\ 0 & 4 \cdot 10^{-4} \end{bmatrix} \text{sec}^{-2} \quad \text{and} \quad \Lambda_d = \begin{bmatrix} 16 \cdot 10^{-3} & 0 \\ 0 & 16 \cdot 10^{-3} \end{bmatrix} \text{sec}^{-1}$$

Statistical performance evaluation

- Relative guidance maneuver phases

- During the merging phase, the purpose of the relative guidance feedback loop is to track the delayed and projected leading aircraft position (position C)
- As soon as the delayed leading aircraft position has passed the merging meter fix, the projection onto the trailing aircraft flight plan is no longer necessary



Statistical performance evaluation

- Encounter scenarios

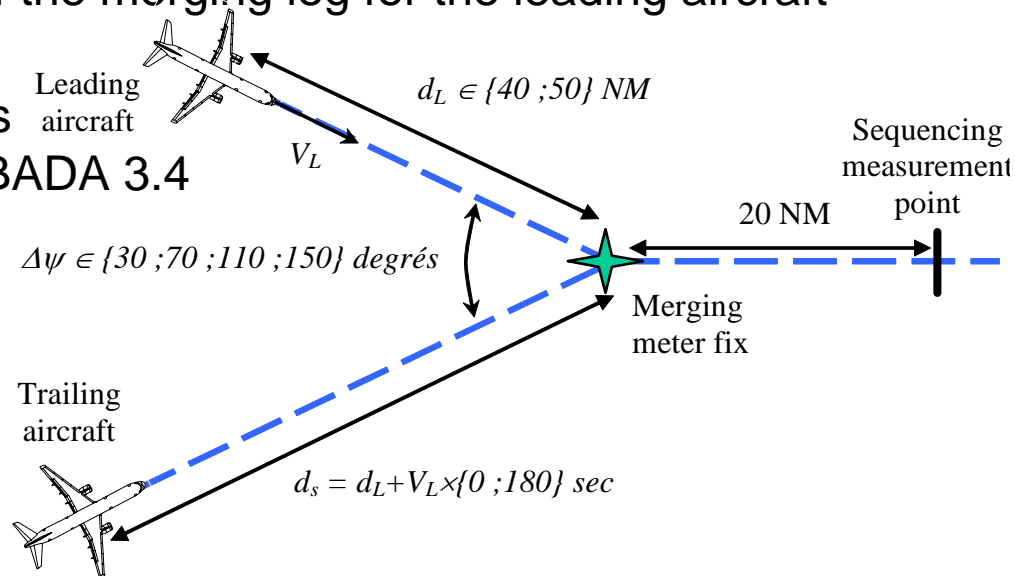
- Encounters have been generated by changing the angle between the two convergence legs, the length of the merging leg for the leading aircraft and the aircraft type

- Six representative aircraft types selected from the Eurocontrol BADA 3.4 aircraft performance database

- ATR42/72, SAAB2000, A320, B767-300, A340, B747-400

- At the beginning of the encounter, same flight level which is set between FL100 and FL260 (insofar it is flyable)

- At the end of the encounter, both aircraft level off at FL100



Statistical performance evaluation

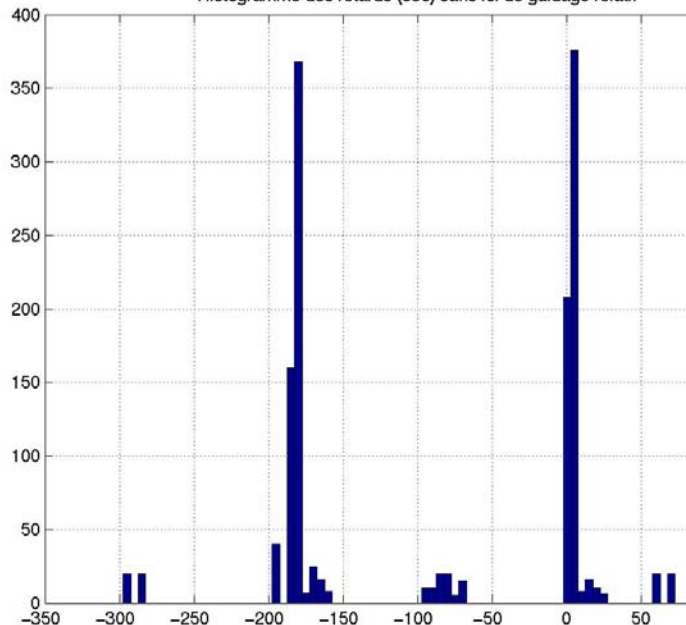
- Encounter scenarios
 - The initial position of the leading aircraft is set at a distance from the merging fix equal to $d_L + V_L \times \{0 ; 180\}$ seconds, where d_L stands for the length of the leading aircraft merging leg and V_L for the initial airspeed of the leading aircraft
 - Insofar the objective for the relative guidance feedback loop is to position the following aircraft 90 seconds behind the leading aircraft, it shall delay or speed up the trailing aircraft by 90 seconds;
 - Scenarios validation
 - The difference between the conventional airspeed of both aircraft is less than 30kts (airspeed compatibility)
 - and the expected distance to achieve the merging maneuver is greater than the actual distance between the initial position of the trailing aircraft and the point where the sequencing is measured
 - These considerations have led to the generation of 1408 scenarios

Statistical performance evaluation

- Delay between the leading and the trailing aircraft at the so-called sequencing measurement point
 - Linear and nonlinear design leads to practically the same time lag
 - This time lag complies with the specified time lag

Without feedback loop

Histogramme des retards (sec) sans loi de guidage relatif



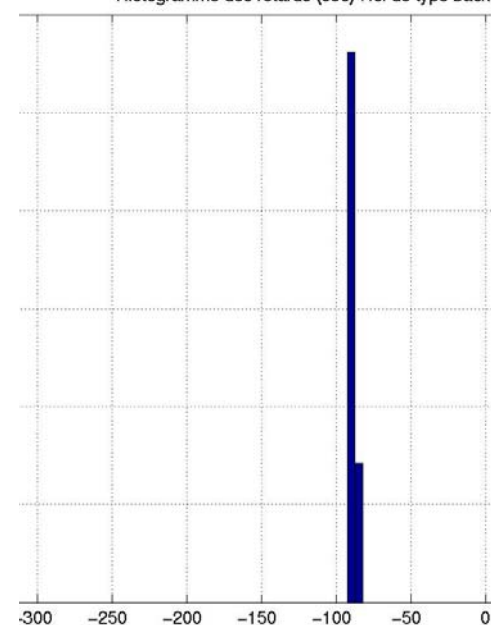
Linear feedback

Histogramme des retards (sec) : loi de type proporti



Nonlinear feedback

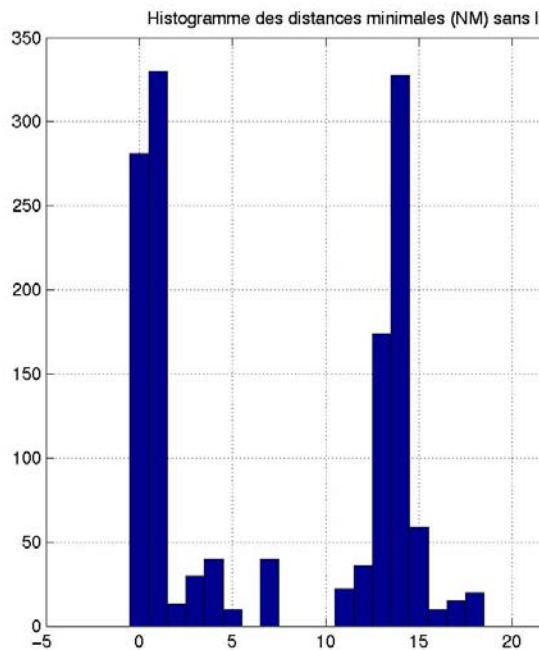
Histogramme des retards (sec) : loi de type backs



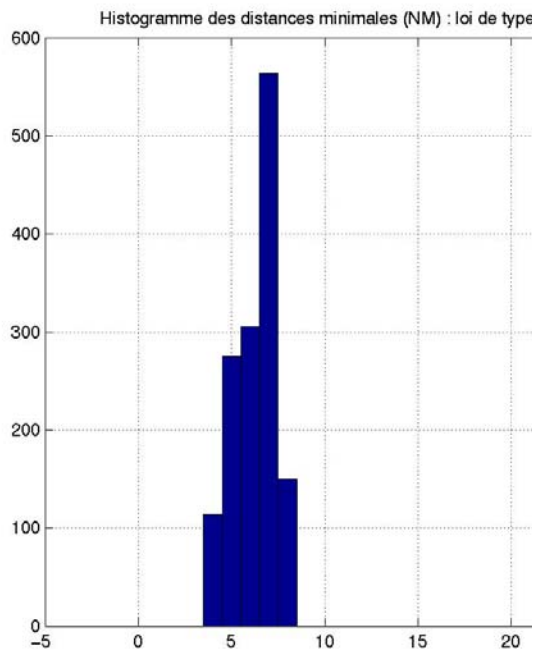
Statistical performance evaluation

- Minimum distance between the two aircraft
 - The nonlinear design is marginally better than the linear design

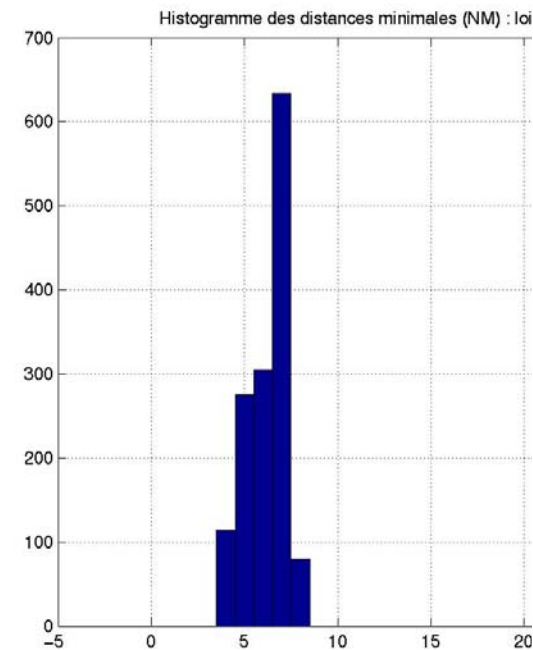
Without feedback loop



Linear feedback



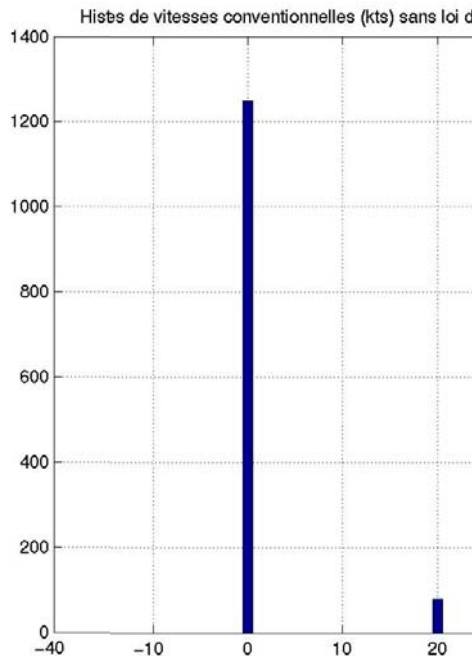
Nonlinear feedback



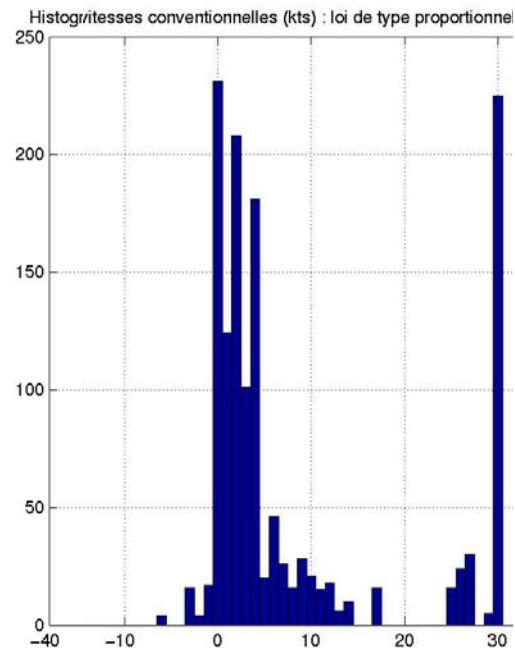
Statistical performance evaluation

- Dispersion of the difference between the conventional airspeed (CAS) of the leading and the trailing aircraft at the sequencing measurement point
 - It is here that the greatest improvement is found

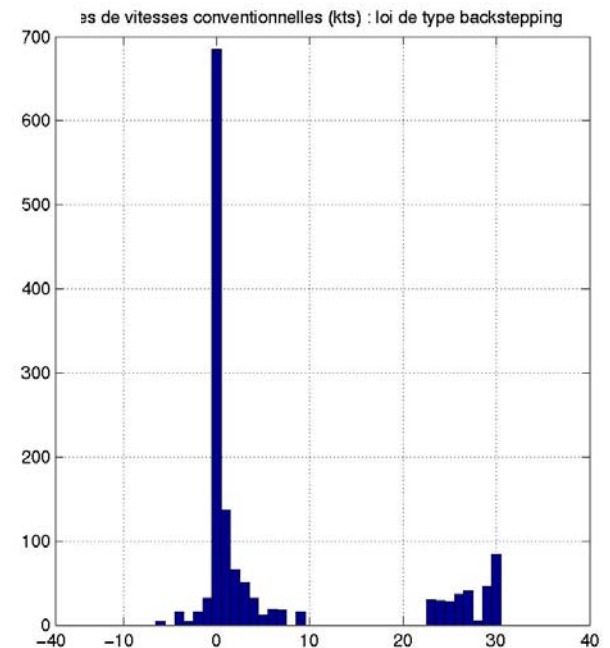
Without feedback loop



Linear feedback



Nonlinear feedback



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Conclusion

- Main points of this paper
 - A new nonlinear design of the feedback control loop based on vectorial backstepping
 - Both linear and nonlinear designs give satisfactory results
 - Nonlinear design brings a greater benefit in speed regulation
- Further refinements and validations
 - Stretching the trajectory of the trailing aircraft through the generation of a reference trajectory for longer delay compensation